EXCERPTED FROM

STEPHEN WOLFRAM
A NEW KIND OF SCIENCE

SECTION 3.4

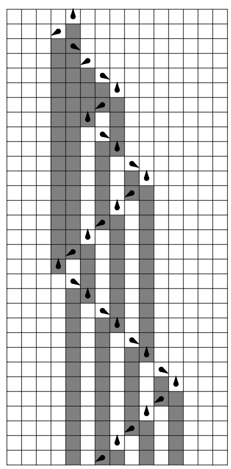
Turing Machines

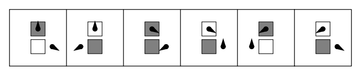
Turing Machines

In the history of computing, the first widely understood theoretical computer programs ever constructed were based on a class of systems now called Turing machines.

Turing machines are similar to mobile automata in that they consist of a line of cells, known as the "tape", together with a single active cell, known as the "head". But unlike in a mobile automaton, the head in a Turing machine can have several possible states, represented by several possible arrow directions in the picture below.

And in addition, the rule for a Turing machine can depend on the state of the head, and on the color of the cell at the position of the head, but not on the colors of any neighboring cells.

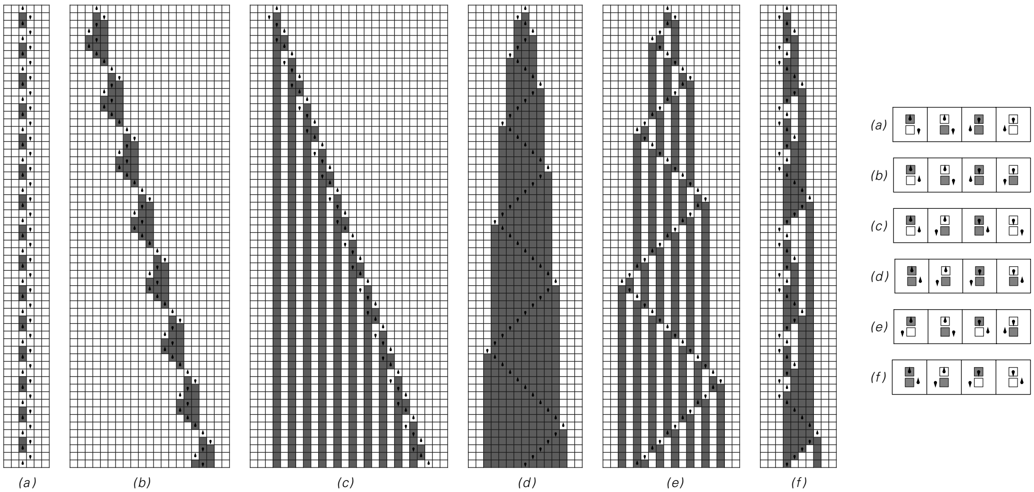




An example of a Turing machine. Like a mobile automaton, the Turing machine has one active cell or "head", but now the head has several possible states, indicated by the directions of the arrows in this picture.

Turing machines are still widely used in theoretical computer science. But in almost all cases, one imagines constructing examples to perform particular tasks, with a huge number of possible states and a huge number of possible colors for each cell.

But in fact there are non-trivial Turing machines that have just two possible states and two possible colors for each cell. The pictures on the facing page show examples of some of the 4096 machines of this kind. Both repetitive and nested behavior are seen to occur, though nothing more complicated is found.



Examples of Turing machines with two possible states for the head. There are a total of 4096 rules of this kind. Repetitive and nested patterns are seen, but nothing more complicated ever occurs.

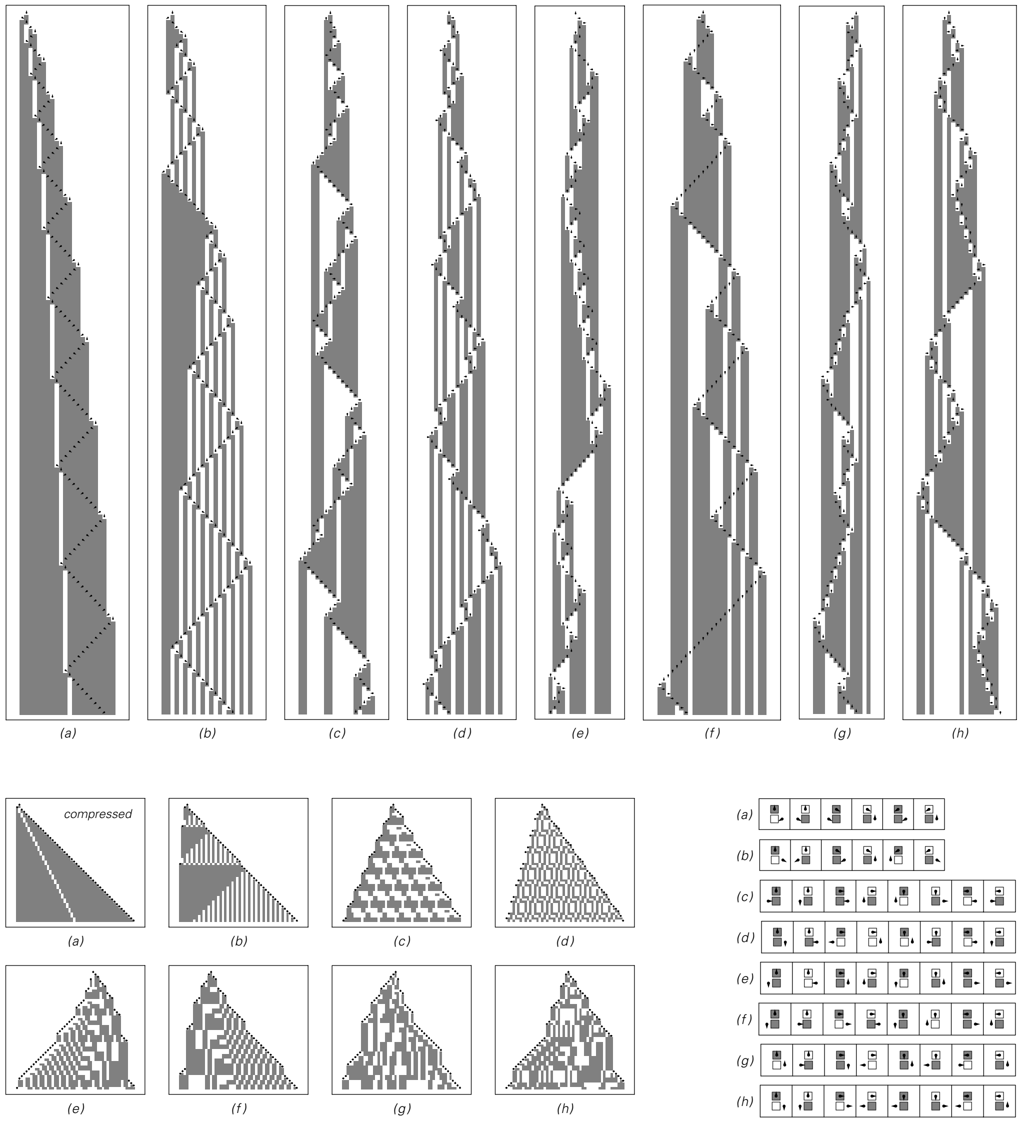
From our experience with mobile automata, however, we expect that there should be Turing machines that have more complex behavior.

With three states for the head, there are about three million possible Turing machines. But while some of these give behavior that looks slightly more complicated in detail, as in cases (a) and (b) on the next page, all ultimately turn out to yield just repetitive or nested patterns—at least if they are started with all cells white.

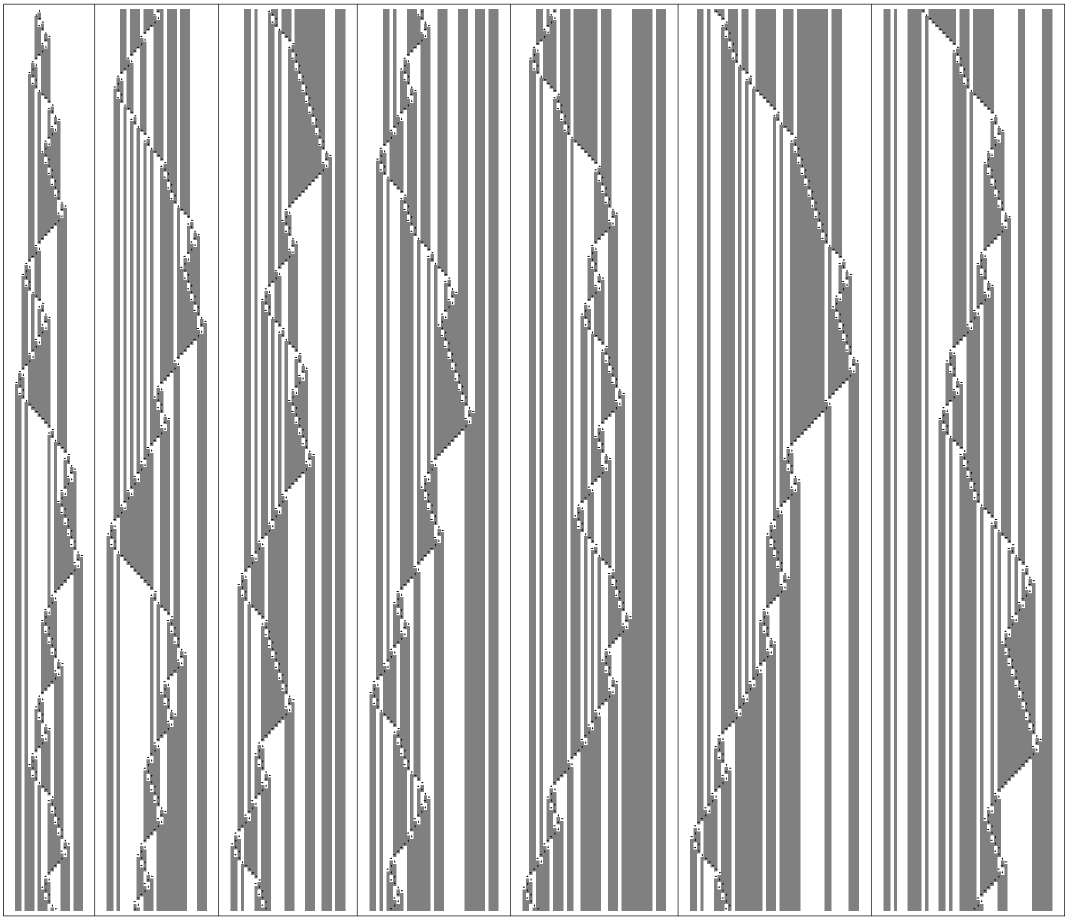
With four states, however, more complicated behavior immediately becomes possible. Indeed, in about five out of every million rules of this kind, one gets patterns with features that seem in many respects random, as in the pictures on the next two pages.

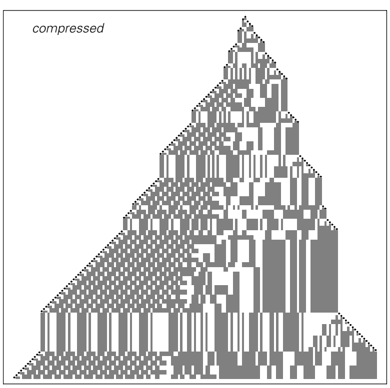
So what happens if one allows more than four states for the head? It turns out that there is almost no change in the kind of behavior one sees. Apparent randomness becomes slightly more common, but otherwise the results are essentially the same.

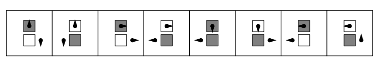
Once again, it seems that there is a threshold for complex behavior—that is reached as soon as one has at least four states. And just as in cellular automata, adding more complexity to the underlying rules does not yield behavior that is ultimately any more complex.



Examples of Turing machines with three and four possible states. With three possible states, only repetitive and nested patterns are ever ultimately produced, at least starting with all cells white. But with four states, more complicated patterns are generated. The top set of pictures show the first 150 steps of evolution according to various different rules, starting with the head in the first state (arrow pointing up), and all cells white. The bottom set of pictures show the evolution in each case in a compressed form. Each of these pictures includes the first 50 steps at which the head is further to the left or right than it has ever been before.







A Turing machine that exhibits behavior which seems in many respects random. The Turing machine has four possible states for its head, and two possible colors for each cell on its tape. It starts with all cells white, corresponding to a blank tape. Each column above shows 250 steps of evolution; the compressed form on the left corresponds to a total of 20,000 steps.